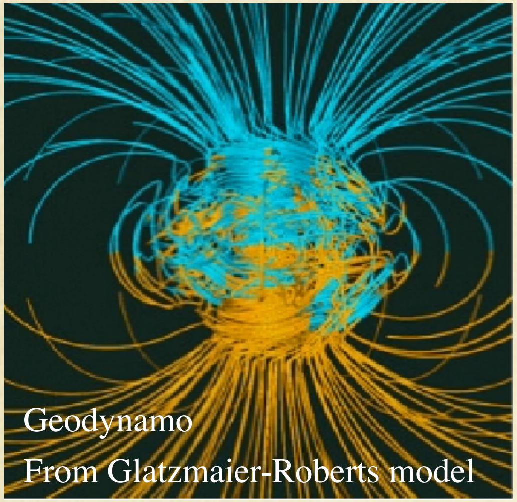
Small-scale Turbulence as Regenerating Source of the Stellar Magnetic Fields

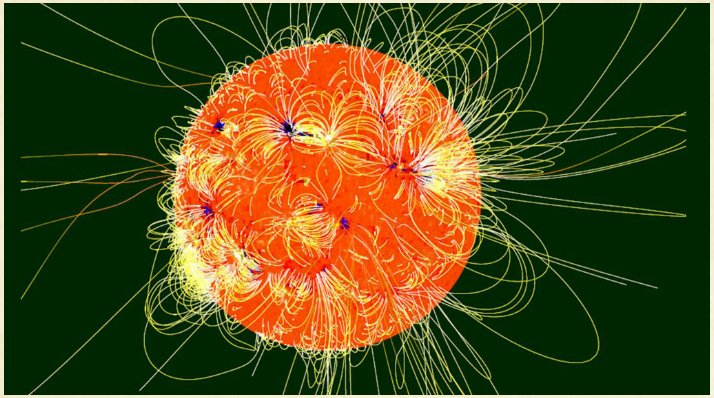
Giuseppina Nigro

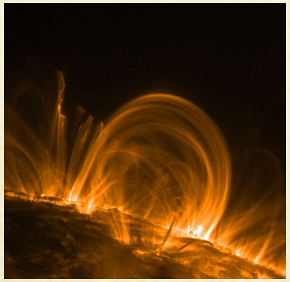
– Physics Department, The University of Rome Tor Vergata

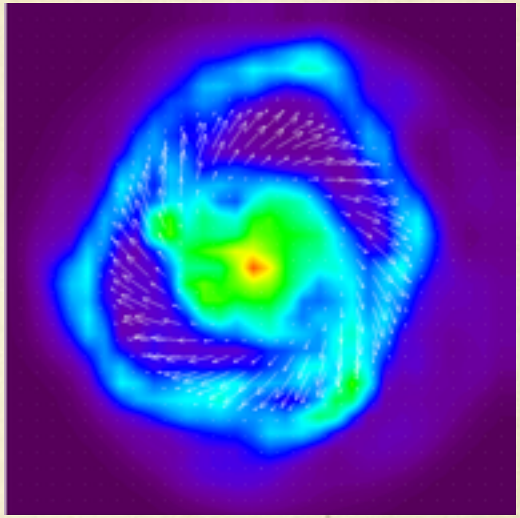
Magnetic Dynamo

The fundamental phenomenon to explain the origin of the magnetic field in astrophysical systems, like planets, stars, interstellar and intergalactic medium, etc.









The generation and the dynamics of a magnetic field is described by the induction equation.
A realistic parameter regime is beyond the power of today's supercomputers (larges Re, Rm)

Magnetic Dynamo

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \frac{1}{4\pi\rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau},$$

Induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}),$$

inductive term + resistive dissipation

$\mathrm{Rm} = \mathrm{uL}/\eta \gg 1$

Magnetic Reynolds number

Large Reynolds numbers => large range of time and space scales

Mean Field Electrodynamics

Scale decomposition: the field is made of a mean component **<u>** + fluctuating part at small scales **u'** (**Not a linearization: |u'|/|<u>| not << 1**)

$$u = \langle u \rangle + u' \qquad \langle u' \rangle = 0$$

$$B = \langle B \rangle + B' \qquad \langle B' \rangle = 0$$

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times \left(\langle \mathbf{u} \rangle \times \langle \mathbf{B} \rangle + \langle \mathbf{u}' \times \mathbf{B}' \rangle - \eta \nabla \times \langle \mathbf{B} \rangle \right),$$

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\alpha} : \langle \mathbf{B} \rangle + \boldsymbol{\beta} : \nabla \times \langle \mathbf{B} \rangle$$

For homogeneous and isotropic turbulence α & β tensors reduce to scalars:

$$\boldsymbol{\mathcal{E}} = \alpha \langle \mathbf{B} \rangle - \eta_{\mathrm{T}} \nabla \times \langle \mathbf{B} \rangle .$$

These quantities (α & η_T) are treated as adjustable parameters in mean-field models => hence large-scale dynamo models become parametric models

$$\alpha \to \alpha(\langle \mathbf{B} \rangle) = \frac{\alpha_0}{1 + (\langle \mathbf{B} \rangle / B_{\mathrm{eq}})^2}.$$

$$\alpha = \mu \left(1 - \frac{b_1^2}{B_0^2} \right),$$

Magnetic Dynamo

$$\frac{\partial \vec{B}_0}{\partial t} = \nabla \times (\vec{u}_0 \times \vec{B}_0) + \nabla \times \vec{\varepsilon} + \eta \nabla^2 \vec{B}_0 \qquad with \qquad \vec{\varepsilon} = \left\langle \delta\vec{u} \times \delta\vec{b} \right\rangle$$

$$\frac{\partial \delta\vec{b}}{\partial t} = (\vec{B}_0 \cdot \nabla)\ \delta\vec{u}\ - (\vec{u}_0 \cdot \nabla)\ \delta\vec{b} + (\delta\vec{u} \cdot \nabla)\ \vec{u}_0 - (\delta\vec{u} \cdot \nabla)\ \vec{B}_0 +$$

$$-\left[\left\langle (\delta\vec{b} \cdot \nabla)\ \delta\vec{u} \right\rangle - (\delta\vec{b} \cdot \nabla)\ \delta\vec{u}\right] + \left\langle (\delta\vec{u} \cdot \nabla)\ \delta\vec{b} \right\rangle - (\delta\vec{u} \cdot \nabla)\ \delta\vec{b}\ - \eta \nabla^2 \delta\vec{b}$$

$$\frac{\partial \delta\vec{u}}{\partial t} = -(\vec{u}_0 \cdot \nabla)\ \delta\vec{u}\ - (\delta\vec{u} \cdot \nabla)\ \vec{u}_0 + \left\langle (\delta\vec{u} \cdot \nabla)\ \delta\vec{u} \right\rangle - (\delta\vec{u} \cdot \nabla)\ \delta\vec{u}\ - (\nabla P - \langle \nabla P \rangle) +$$

$$-\frac{1}{4\pi\rho}\left[(\vec{B}_0 \cdot \nabla)\ \delta\vec{b}\ + (\delta\vec{b} \cdot \nabla)\ \vec{B}_0 + (\delta\vec{b} \cdot \nabla)\ \delta\vec{b}\ - \left\langle (\delta\vec{b} \cdot \nabla)\ \delta\vec{b} \right\rangle\right] + \nu \nabla^2 \delta\vec{u}$$

Shell Models

$$\left(\frac{\partial}{\partial t} + \nu k^2\right) u_\mu(\mathbf{k}, t) = \sum_p M_{\mu\alpha\beta}\, u_\alpha(\mathbf{k} - \mathbf{p}, t)\, u_\beta(\mathbf{p}, t)$$

$$M_{\mu\alpha\beta} = \frac{1}{2i}(D_{\mu\alpha}k_\beta + D_{\mu\beta}k_\alpha)\ , \qquad D_{\mu\alpha} = \left(\delta_{\mu\alpha} - \frac{k_\mu k_\alpha}{k^2}\right) \qquad \mu, \alpha, \beta = 1,2,3$$

1) Introduce an exponential spacing of the wave vectors space (shells)

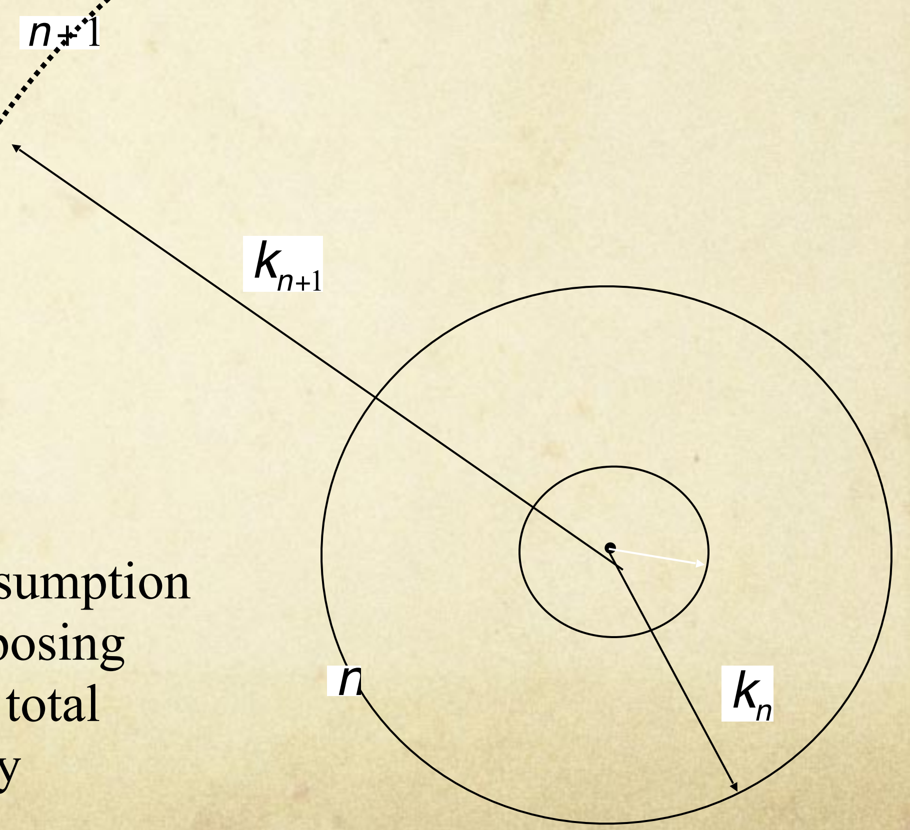
$$l_0 = 2\pi / k_0$$

$$k_n = k_0 2^n$$
$$n = 1,2,\ldots, N$$

2) Assign to each shell dynamical variables

$$u_n(t) \quad b_n(t)$$

3) Nonlinear terms are written under the assumption that interaction in k-space are local and imposing that they conserve the quadratic invariants: total energy, cross helicity, and magnetic helicity



A Shell Model for Turbulent Dynamo

The action of small scales on large scales: e.m.f. **->**

$$\overline{\varepsilon} = -\left\langle \sum_{\bar{k}} \overline{u}(\bar{k},t) \times \overline{b}(\bar{k},t) \right\rangle$$

$$\frac{dB_\phi}{dt} = \frac{B_p V}{L} - \eta \frac{B_\phi}{L^2} + i\sum_n \frac{1}{L}\left(u_n^* b_n - u_n b_n^*\right)$$

$$\frac{dB_p}{dt} = -\eta \frac{B_p}{L^2} + i\sum_n \frac{1}{L}\left(u_n^* b_n - u_n b_n^*\right)$$

A Shell Model for Turbulent Dynamo

At large scale the electromotive force is in a form consistent with the shell model and the spatial derivative associated with the large scale is estimated dividing by the typical large scale L:

$$\frac{dB_\phi}{dt} = \frac{B_p V}{L} - \eta \frac{B_\phi}{L^2} + i\sum_n \frac{1}{L}\left(u_n^* b_n - u_n b_n^*\right) \qquad \frac{dB_p}{dt} = -\eta \frac{B_p}{L^2} + i\sum_n \frac{1}{L}\left(u_n^* b_n - u_n b_n^*\right)$$

$$\frac{du_n}{dt} = ik_n\left[\left(u_{n+1}^* u_{n+2} - b_{n+1}^* b_{n+2}\right) - \frac{1}{4}\left(u_{n-1}^* u_{n+1} - b_{n-1}^* b_{n+1}\right) + \frac{1}{8}\left(u_{n-2} u_{n-1} - b_{n-2} b_{n-1}\right)\right] + $$

$$+ ik_n\left(B_\phi + B_p\right)b_n - \nu k_n^2 u_n + f_n$$

$$\frac{db_n}{dt} = ik_n \frac{1}{6}\left[\left(u_{n+1}^* b_{n+2} - b_{n+1}^* u_{n+2}\right) + \left(u_{n-1}^* b_{n+1} - b_{n-1}^* u_{n+1}\right) - \left(u_{n-2} b_{n-1} - b_{n-2} u_{n-1}\right)\right] +$$

$$+ ik_n\left(B_\phi + B_p\right)u_n - \eta k_n^2 b_n$$

Correlated Gaussian noise on the first 3 shells

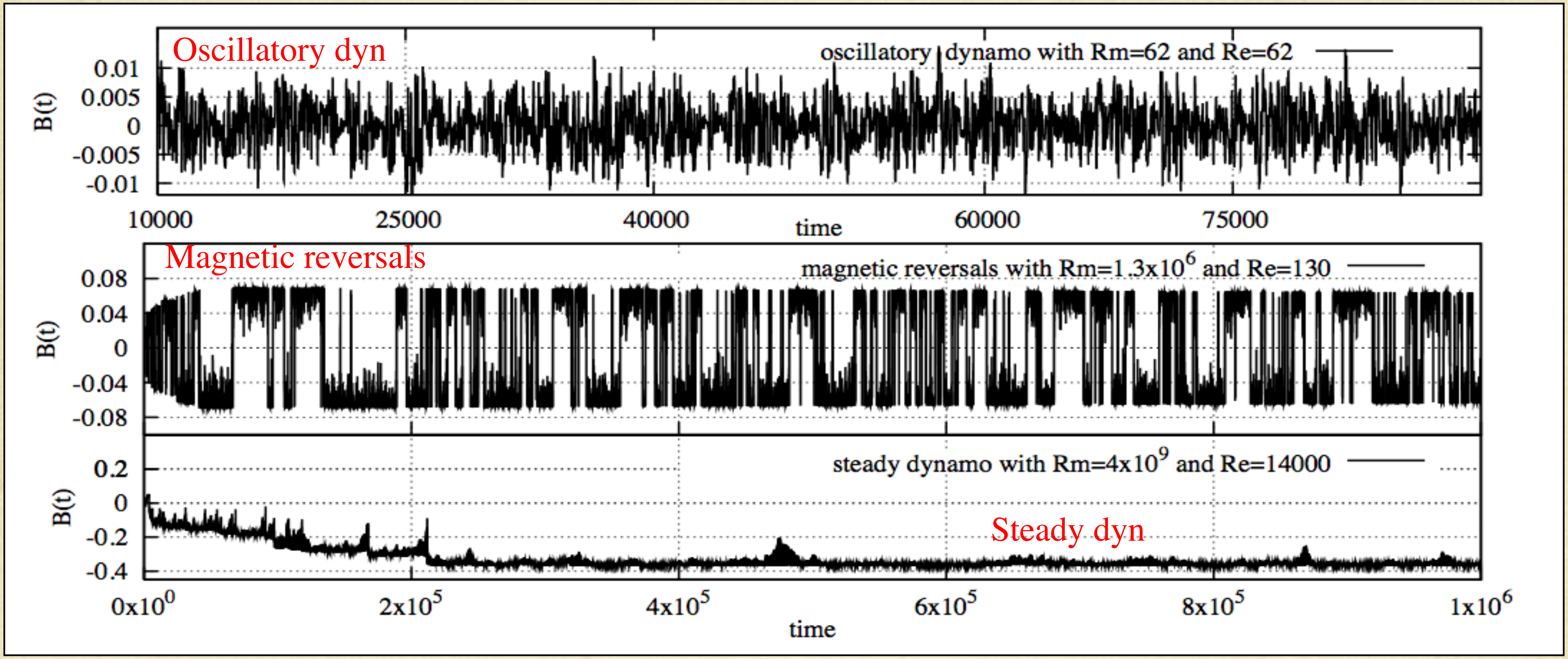
$$\langle f_1^{\,2} \rangle = \sigma^2 / \ln 10 \qquad \sigma = 9 \times 10^{-3}$$

$$k_0 L = 10$$

$V = 0$ **=> Only α-effect => α²-dynamo**

Numerical Results

$\nu = 10^{-5}$



Rm_c=60

G. Nigro, P. Veltri, **740**, L37, ApJL (2011)

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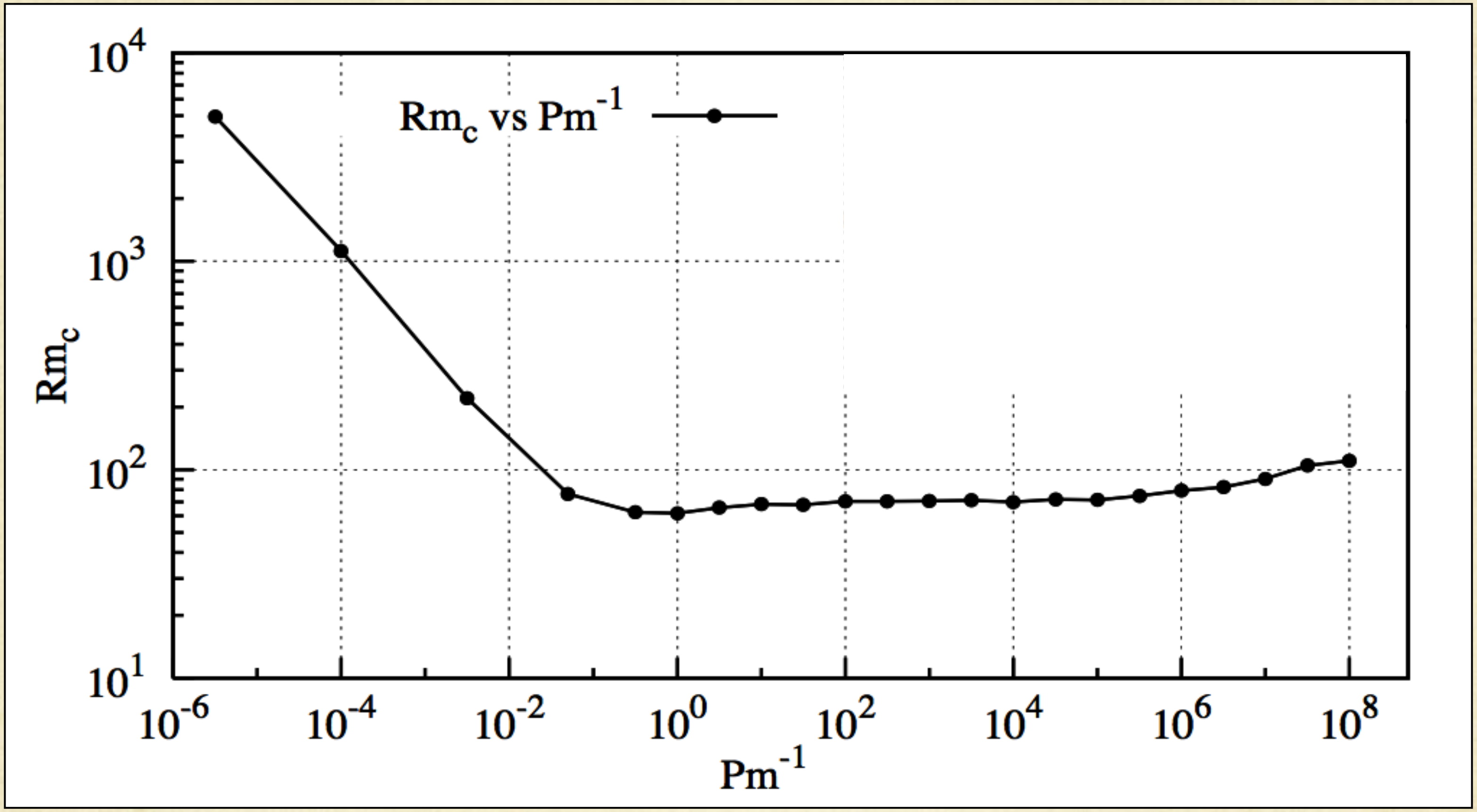
Critical Rm vs Pm^{-1}

$Pm = Rm/Re$



The stability curve Rmc vs Pm^{-1} in log scale. The inset in semi–log scale shows the slight increase of Rmc for increasing $Pm^{-1} > 1$.

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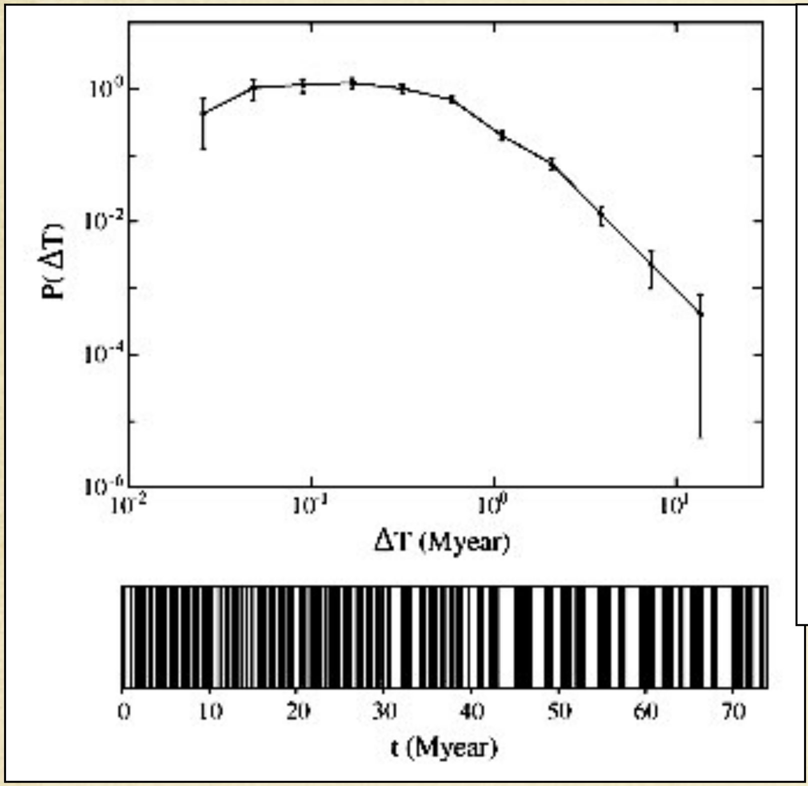
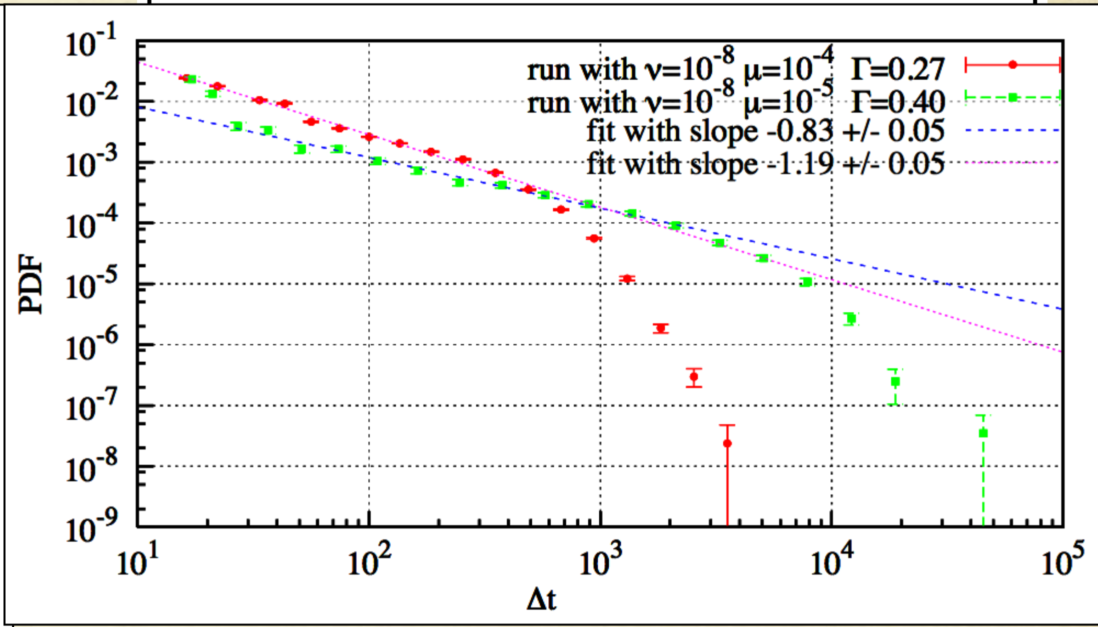
PDF of persistence times

Paleomagnetic Data CK95



Sorriso-Valvo, et al PEPI 164, 197-207 (2007).

Model Result: $\Gamma = \mathrm{rms(B)}/\delta u$



The tendency of the system to develop longer persistent times for increasing large-scale magnetic field strength (see F. Stefani and .Gerbeth, PRL 94, 184506 (2005) and J.A. Tarduno et al., Science 291, 1779 (2001))

PDF displays a power law behavior => non Poisson process=> phenomena characterized by memory effects due to presence of long-range correlation

A Thermally Driven Shell Model for Magneto-convective Dynamo

$$\left(\frac{d}{dt} + \nu k_n^2\right)u_n = -\tilde{\alpha}\theta_n + ik_n\Big[(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) - \frac{\epsilon}{2}(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) - \frac{1-\epsilon}{4}(u_{n-2}u_{n-1} - b_{n-2}b_{n-1})\Big]^* \quad (1)$$

$$\left(\frac{d}{dt} + \eta k_n^2\right)b_n = ik_n\Big[(1-\epsilon-\epsilon_m)(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) + \frac{\epsilon_m}{2}(u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) + \frac{1-\epsilon_m}{4}(u_{n-2}b_{n-1} - b_{n-2}u_{n-1})\Big]^* \quad (2)$$

$$\left(\frac{d}{dt} + \chi k_n^2\right)\theta_n = ik_n[\alpha_1 u^*_{n+1}\theta^*_{n+2} + \alpha_2 u^*_{n+2}\theta^*_{n+1} + \beta_1 u_{n-1}\theta_{n+1} - \beta_2 u_{n+1}\theta_{n-1} + \gamma_1 u_{n-1}\theta_{n-2} + \gamma_2 u_{n-2}\theta_{n-1}]^* + f_n\,, \quad (3)$$

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(GOY) MHD Shell Model

$\epsilon = 1/2 \qquad \epsilon_m = 1/3$

Jensen et al. (PRA, 1992)'s coupling model

$\alpha_1 = \alpha_2 = 1$

$\beta_1 = \beta_2 = 1/2$

$\gamma_1 = \gamma_2 = -1/4$

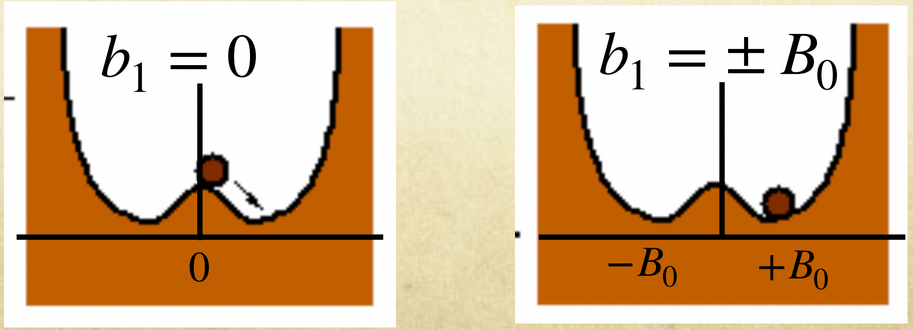
α2 dynamos => We Modified the equation for the largest scale magnetic field

$$\alpha = \mu\left(1 - \frac{b_1^2}{B_0^2}\right)$$

Evolution equation of the large-scale magnetic field:

$$\frac{db_1}{dt} = -\eta k_1^2 b_1 + i\frac{k_1}{6}\left(u_2^* b_3 - b_2^* u_3\right) + \mu b_1\left(1 - \frac{b_1^2}{B_0^2}\right)$$

pitchfork bifurcation

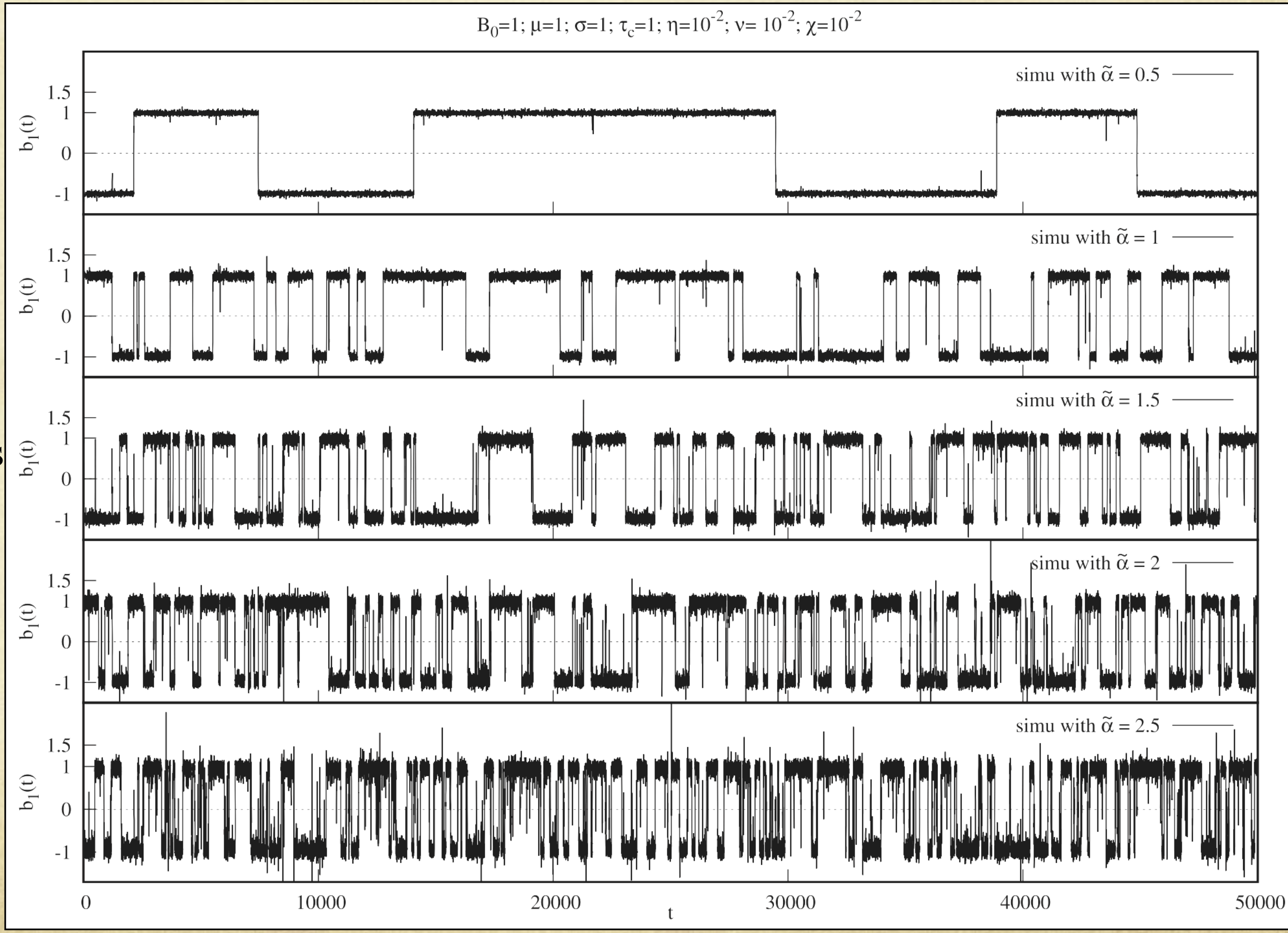


Higher turbulent convection levels make the system more inclined to invert the polarity.

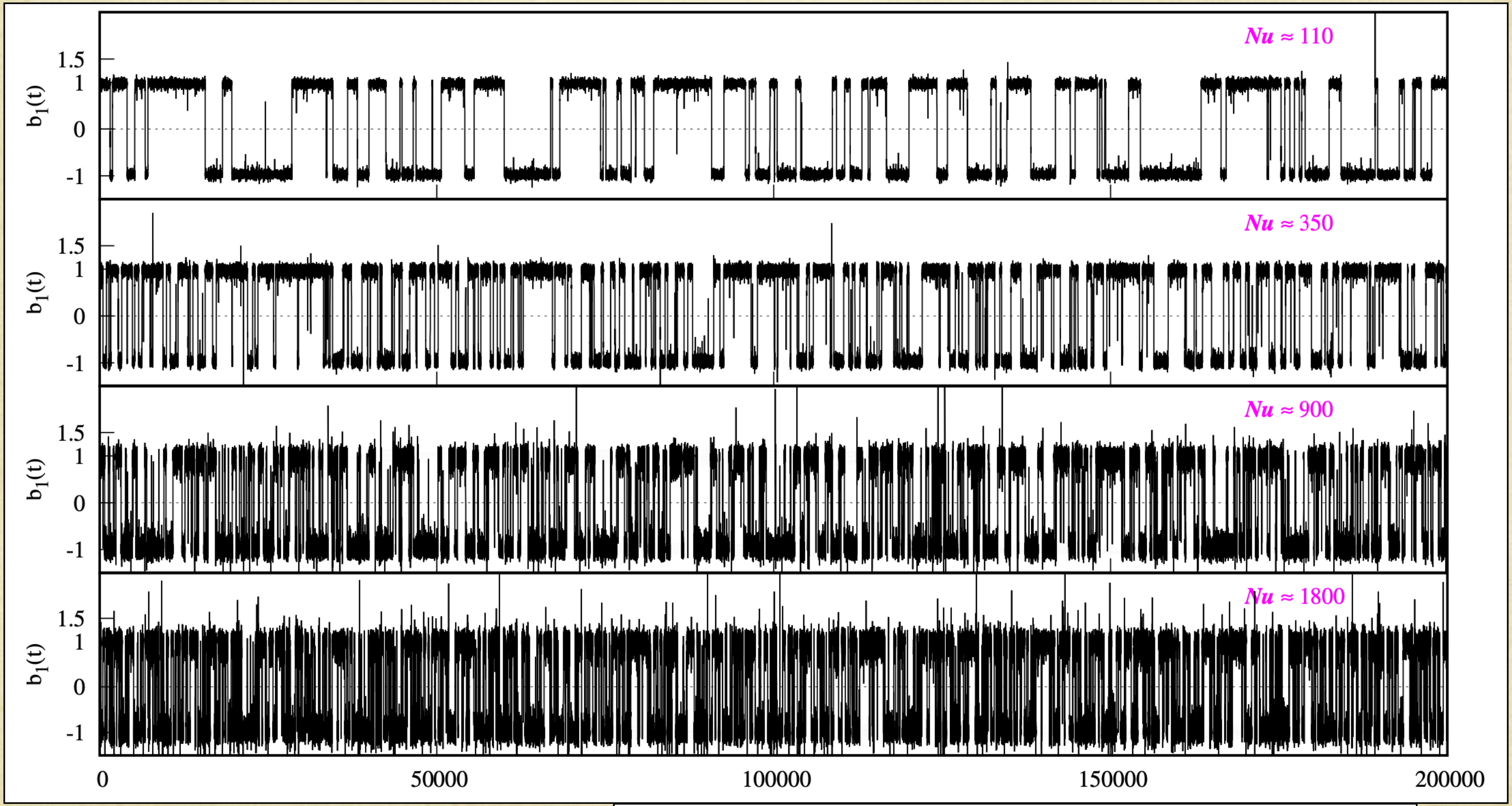
The higher the Rayleigh number

$$Ra = \frac{\tilde{\alpha}\theta_0 L^3}{\nu\chi},$$

the greater the number of reversals



Simulations with higher *Nu* tend to develop much more reversals than those with lower *Nu*



The Nusselt number in the modified shell model:

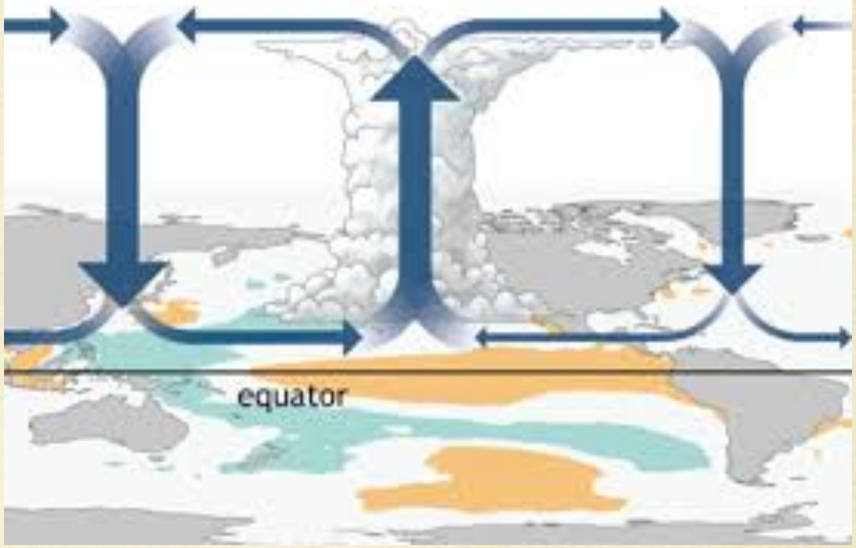
$$Nu = \sqrt{\mathrm{Ra\,Pr}}\,\langle\sum_{n=1}^{N}[u_n(t)\theta_n(t)^* + u_n(t)^*\theta_n(t)]\rangle_t$$

Instantaneous Nusselt Number in RB convection

Nu(t) exhibits instantaneous overshoot above its average value during large-scale circulation reversals (Xi et al. 2016 and Xu et al. 2020)

More coherent flow and plumes that increase heat transfer efficiency

Convective wind of the Earth's atmosphere



Xi et al. 2016 argued that the momentary overshooting behavior in *Nu(t)* could be the **distinguishing feature of the flow reversals among cessations**.

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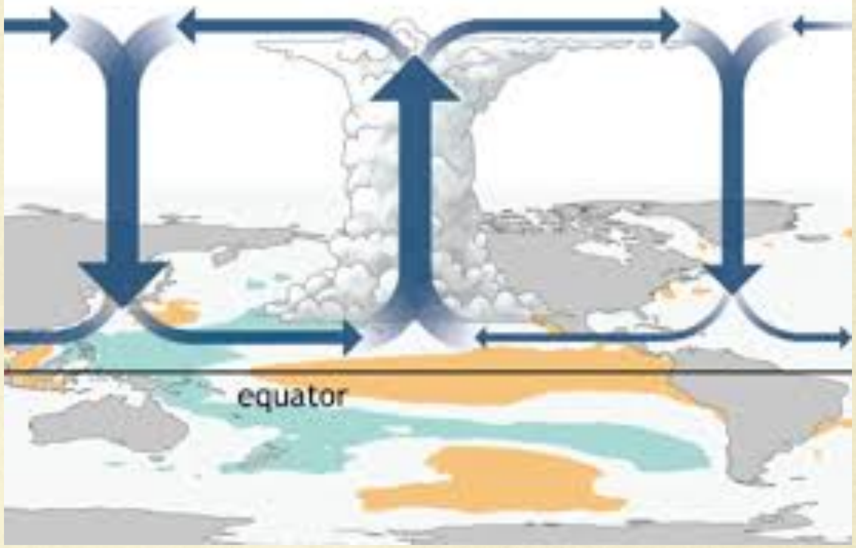
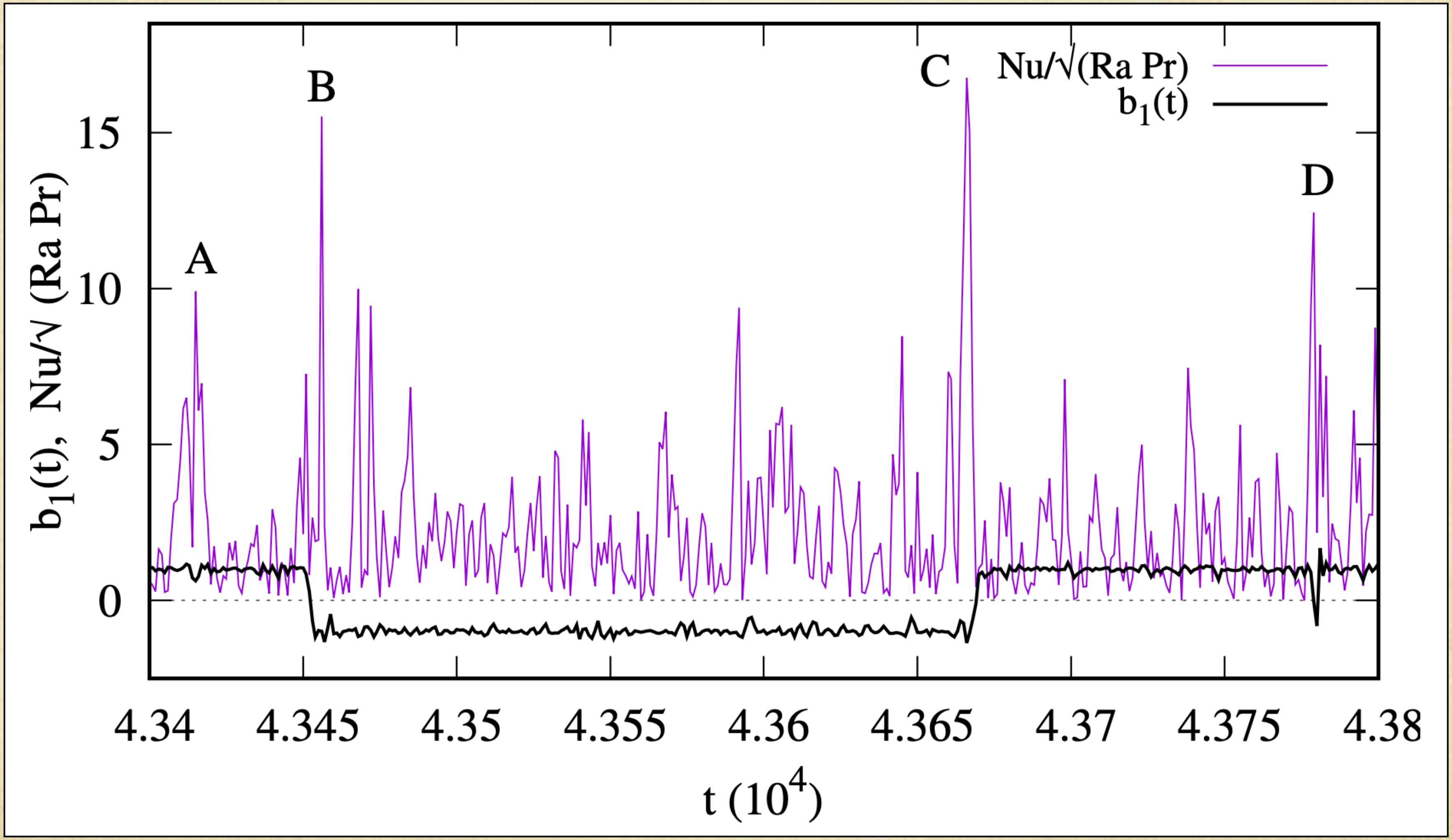


Similarities between large-scale flow reversals in the RB paradigm and magnetic reversals in dynamo (Gallet et al. GAFD 2012, Chandra & Verma PhRvL 2013)

During magnetic polarity reversals, does *Nu(t)* exhibit instantaneous overshoot above its average value?

Highest Nu(t) peaks during magnetic reversals

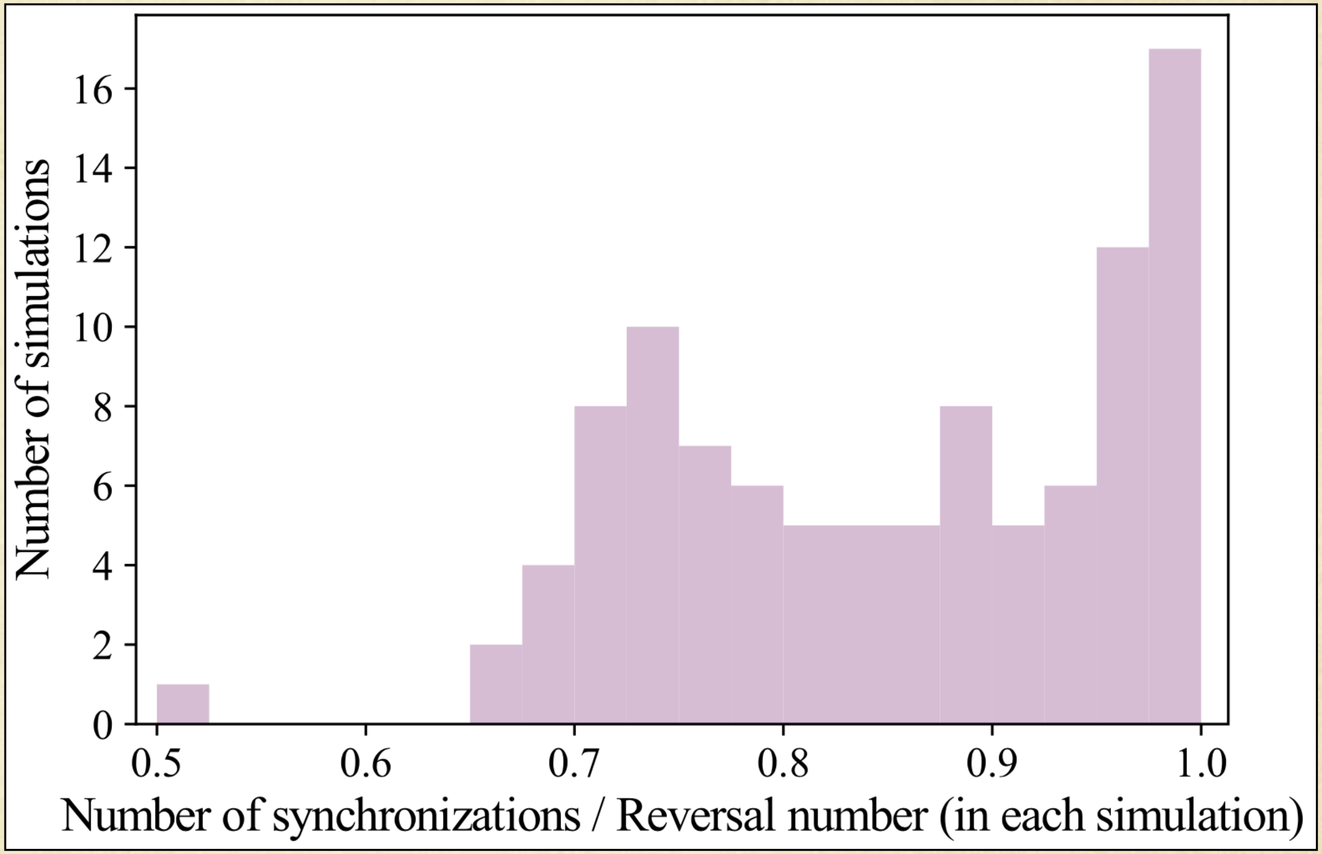
$\chi = \eta = \nu = 10^{-4}$ $\tilde{\alpha} = 0.5$



evaluating a peak as $Nu(t)/\sqrt{\mathrm{Ra\ Pr}} \geqslant \mathrm{C_{thr}}$ $= < Nu > + 2\,\sigma$

Instantaneous Nusselt Number in $\alpha 2$-dynamo

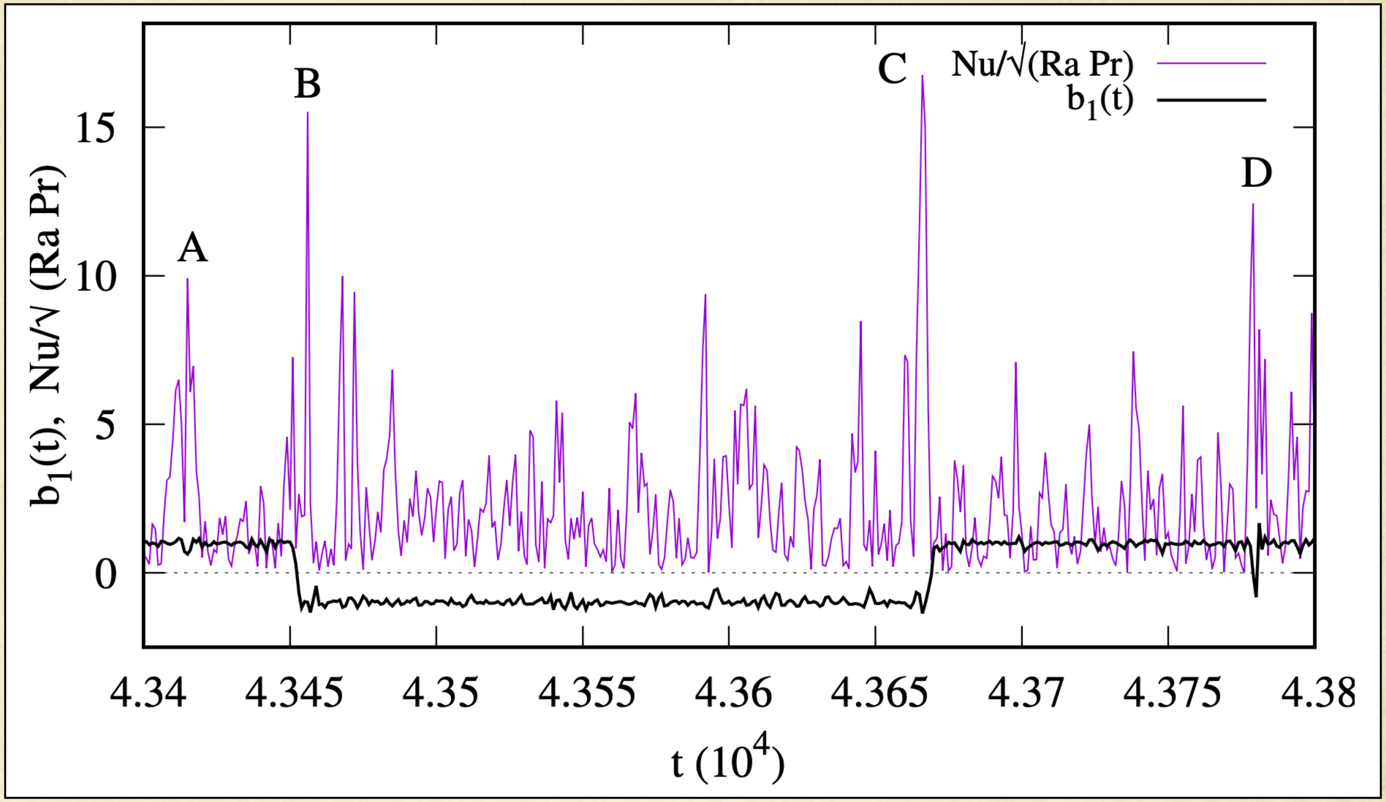
Most of the simulations have a higher than 70% probability that a reversal occurs during a Nu(t) peak.



Synchronisation probability

The causal relationship between Nu(t) peaks and magnetic reversals

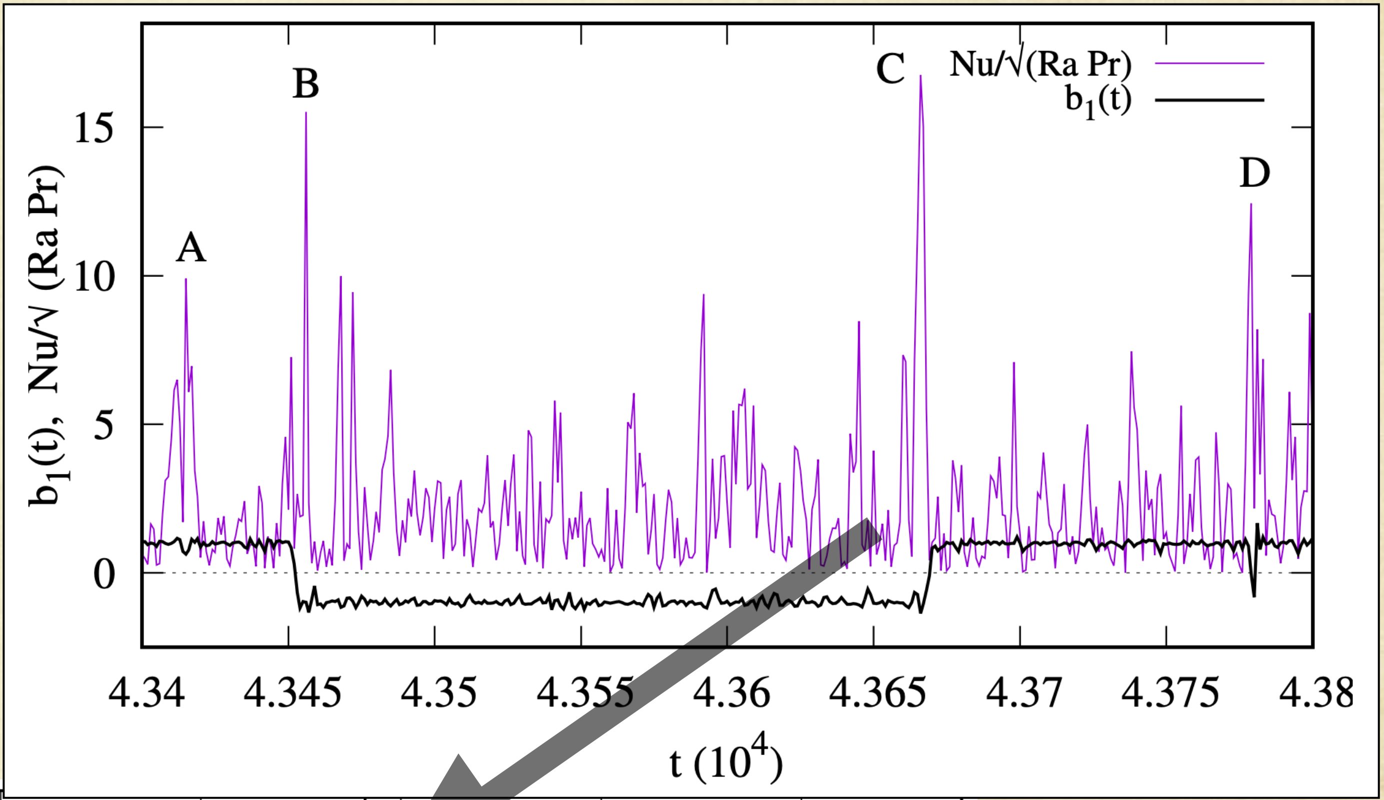
$\chi = \eta = \nu = 10^{-4}$ $\tilde{\alpha} = 0.5$

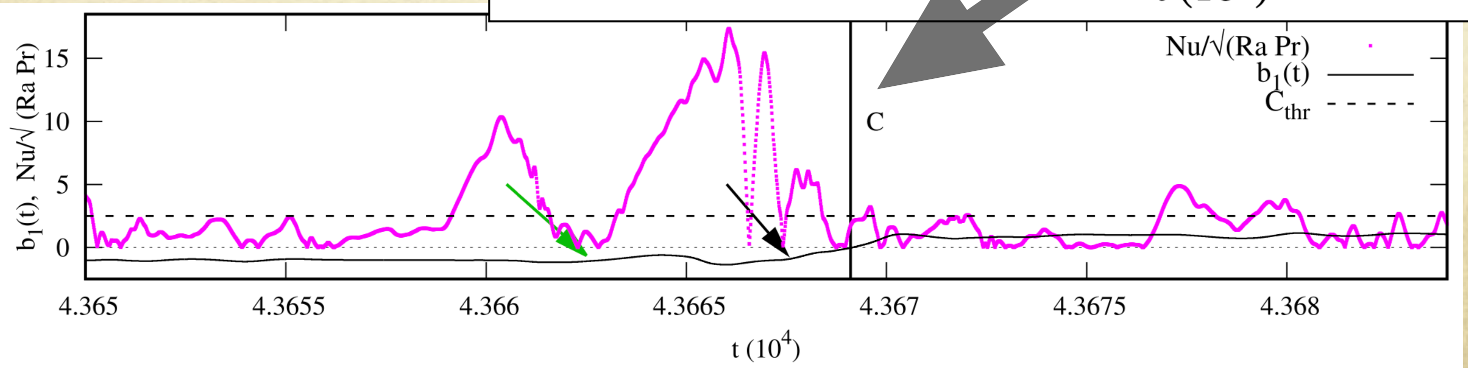


Berkeley (1710) pointed out that correlation does not necessarily imply causation.

A systematic temporal antecedence of a Nu(t) maximum to a reversal can prove that the former could promote the latter's occurrence.

Temporal Antecedence





Convergent Cross-Mapping Analysis

$X = Nu(t)/\sqrt{Ra\,Pr}$ and $Y = db_1(t)/dt.$

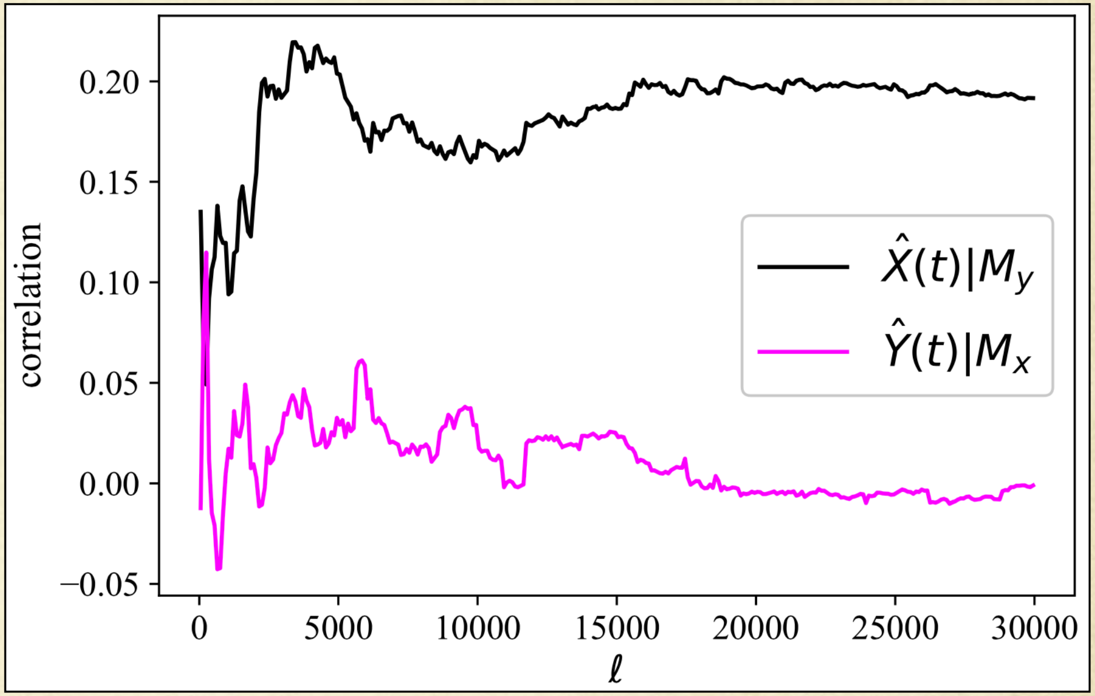
$\chi = \eta = \nu = 10^{-4}$ and $\tilde{\alpha} = 0.5,$

X => Y

If X causes Y, information from X gets embedded in Y. We construct the showdown manifold Mx and My using lagged information of the two-time series, X and Y, respectively. We can use My to predict X, being *X|My* this prediction.

The accuracy of these predictions is thus adopted, in terms of correlation, as a metric for causality.

If the prediction skill of X increases and saturates **as the entire My is used,** this provides evidence that X is causally influencing Y.



CCM analysis shows that **Nu(t) plays a causal role in the magnetic field variation Y**, as indicated by the growth rate in the estimation skill, i.e., correlation, of cross-mapping as **the time series length increases**

Conclusion:
The role of the convective heat flux is important for the reversal occurrence

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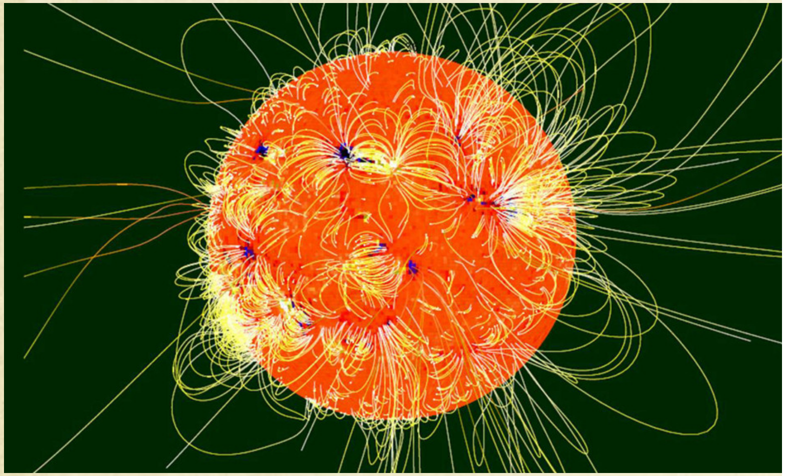
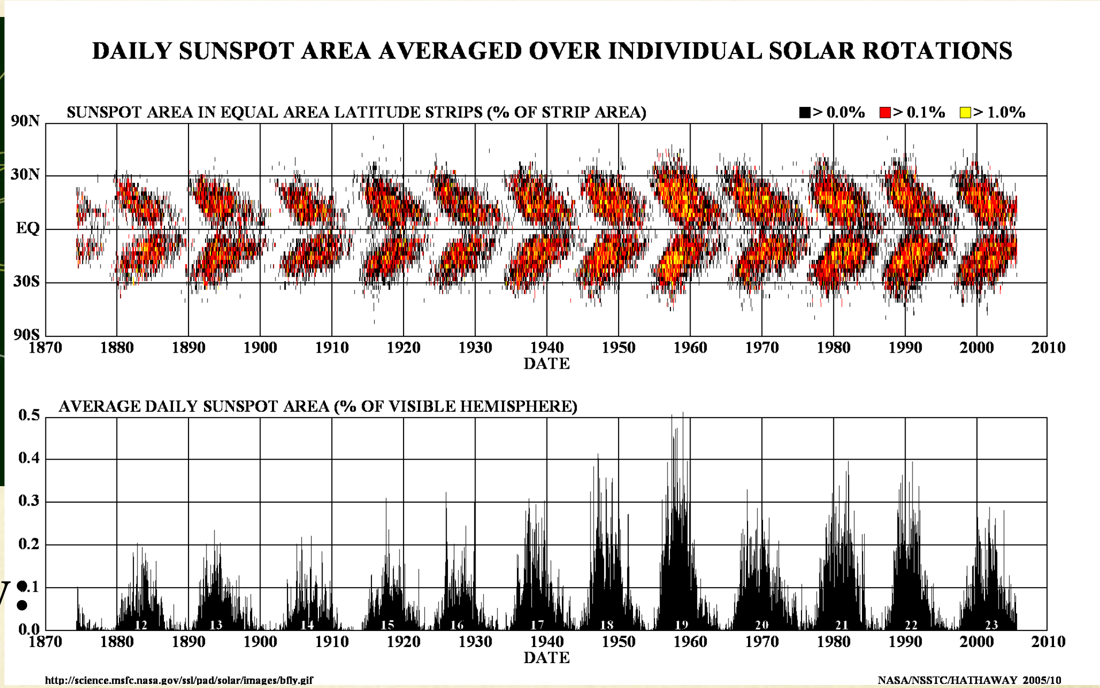
Magnetic Field of the Sun

In Space: large-scale structure



In Time: Coherence (i.e. 11-years cycle)



Large-scale Dynamo Theory:

Advantages:

- Filtering turns an equation with rapidly varying coefficients into ones with smoothly varying coefficients (easier to solve)
- Filtered eqs. are free of the anti-dynamo theorem

Problems:

- A given filtering may not be enough to control the fluctuations
- Do the solution of the filtered equations coincide with the filtered solution of the full equations??

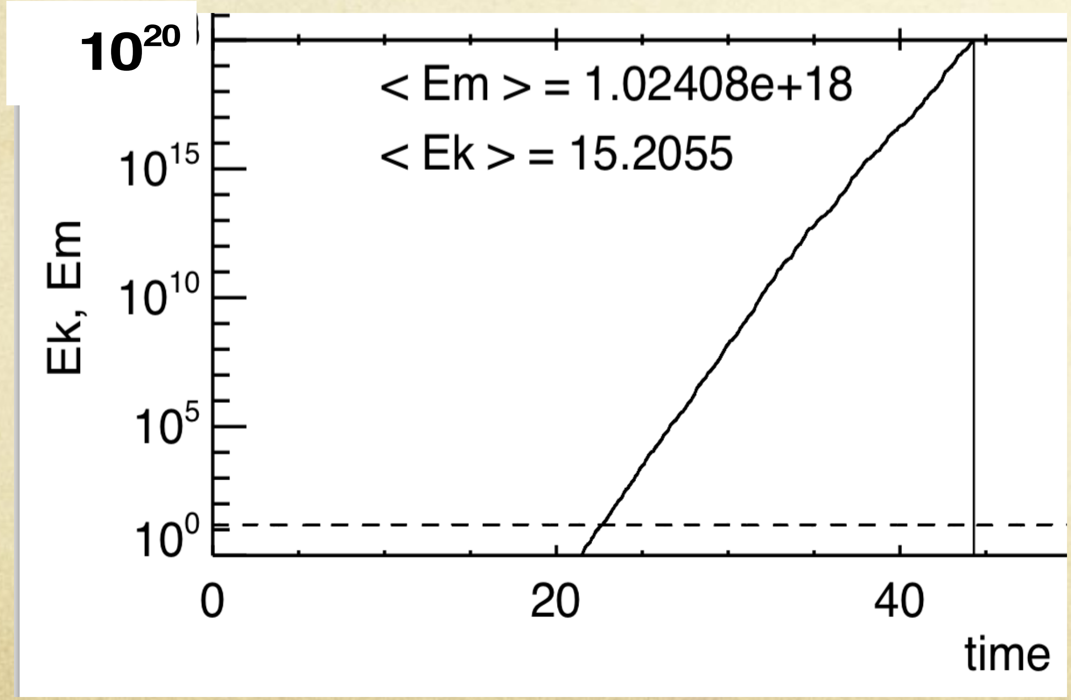
What is a Large-scale Dynamo?

$$\frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{u} \times \bar{B}) - \frac{1}{Rm} \nabla^2 \bar{B}$$

$$\bar{u} = V_0 \cos\left(\frac{2\pi}{L_y} y\right) \hat{e}_x + \textit{helical flow}$$

shear amplitude

$$\mathbf{B} = \mathbf{b}(x, y, t)\, e^{ik_z z}$$



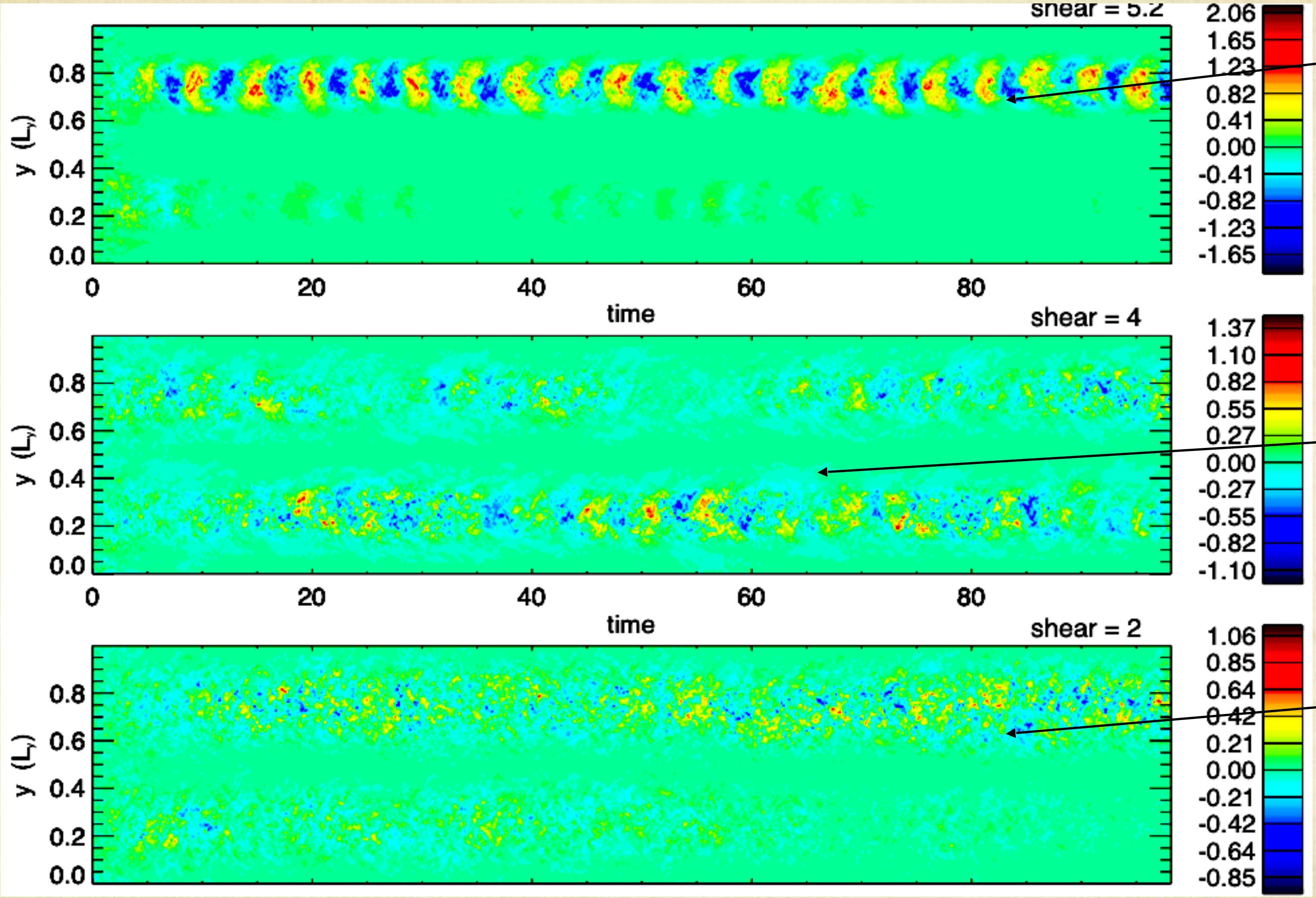
Case when the shear = **5.2**

Growth rate σ, eddy-turn over time τ:

$\sigma = \mathbf{1.0}$ ^ $\tau = \mathbf{0.2}$

All components (at small and large scale) grow at the same rate: **This rate is determined by the small-scales that have been removed from the filtered equations**

B_x wave component when kx=0



Wave for kx =0; what about the other wave number kx≠ 0 ?

Wave ?

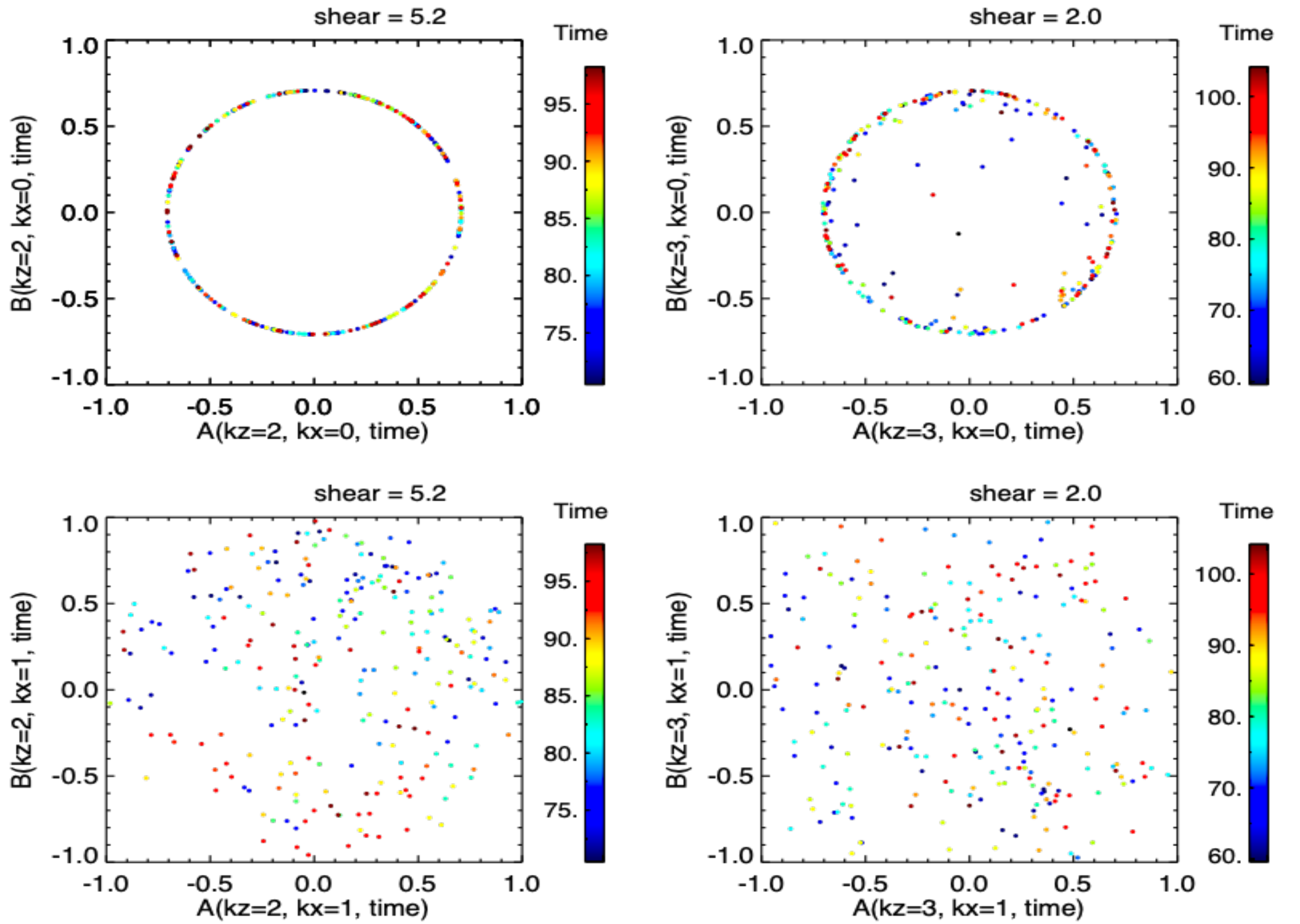
I do not distinguish any wave, but may be…

G. Nigro, P. Pongkitiwaichakul, F. Cattaneo, S.M. Tobias, **464,** L119–L123 (2017) **MNRAS**

P. Pongkitiwaichakul , **G. Nigro** , F. Cattaneo , and S. M. Tobias, **825,** 23 (2016) **ApJ**

Phase Diagrams

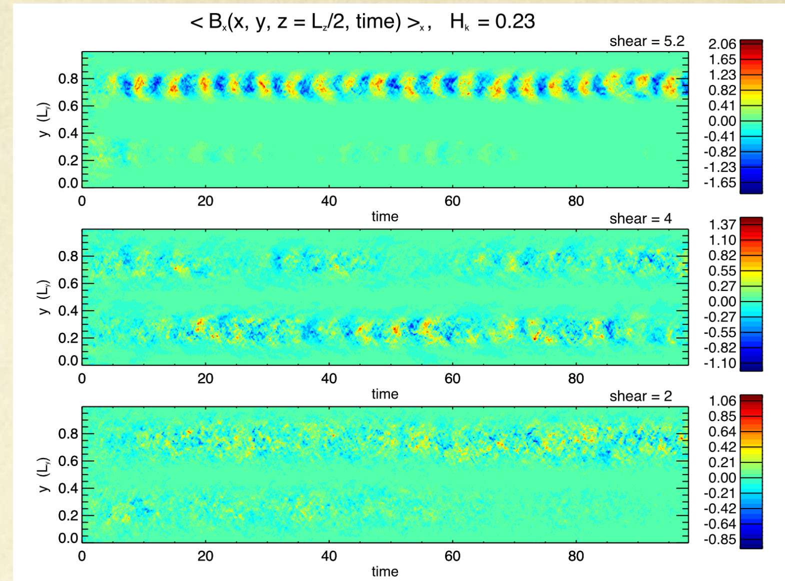
$$\text{scale } 1/kx \Rightarrow \quad FT_x\left[B_x\ (x,y,z,t)\right] = \left[A_{kx}(y,\ t)\ \sin(k_z z) + B_{kx}(y,\ t)\ \cos(k_z z)\right]\ e^{\sigma t}$$



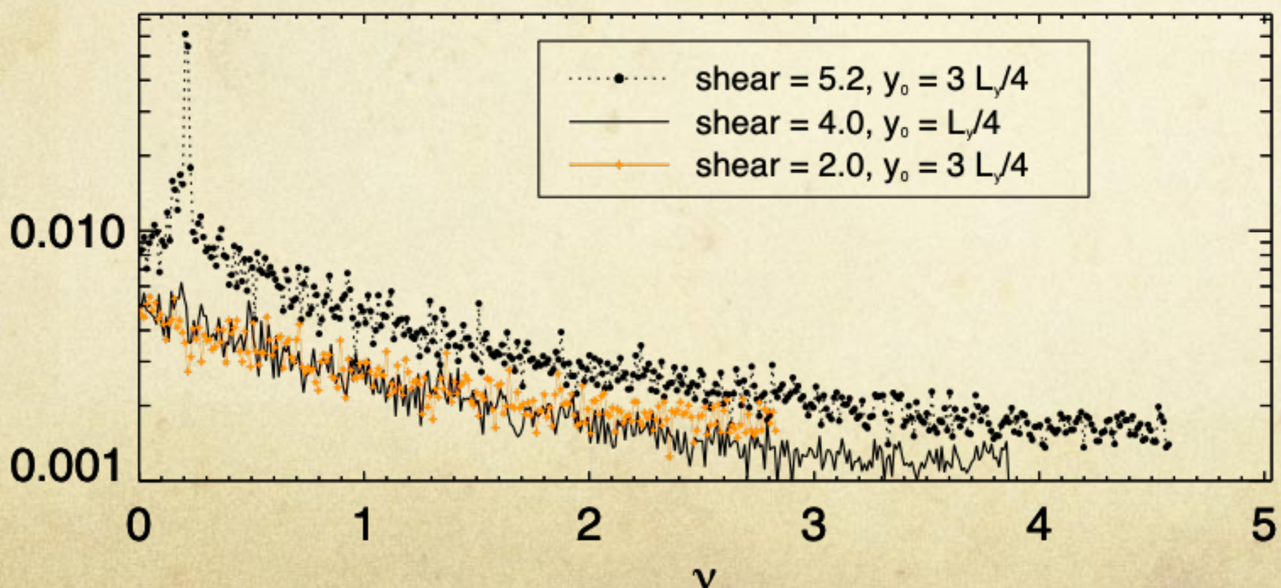
Frequencies of the waves

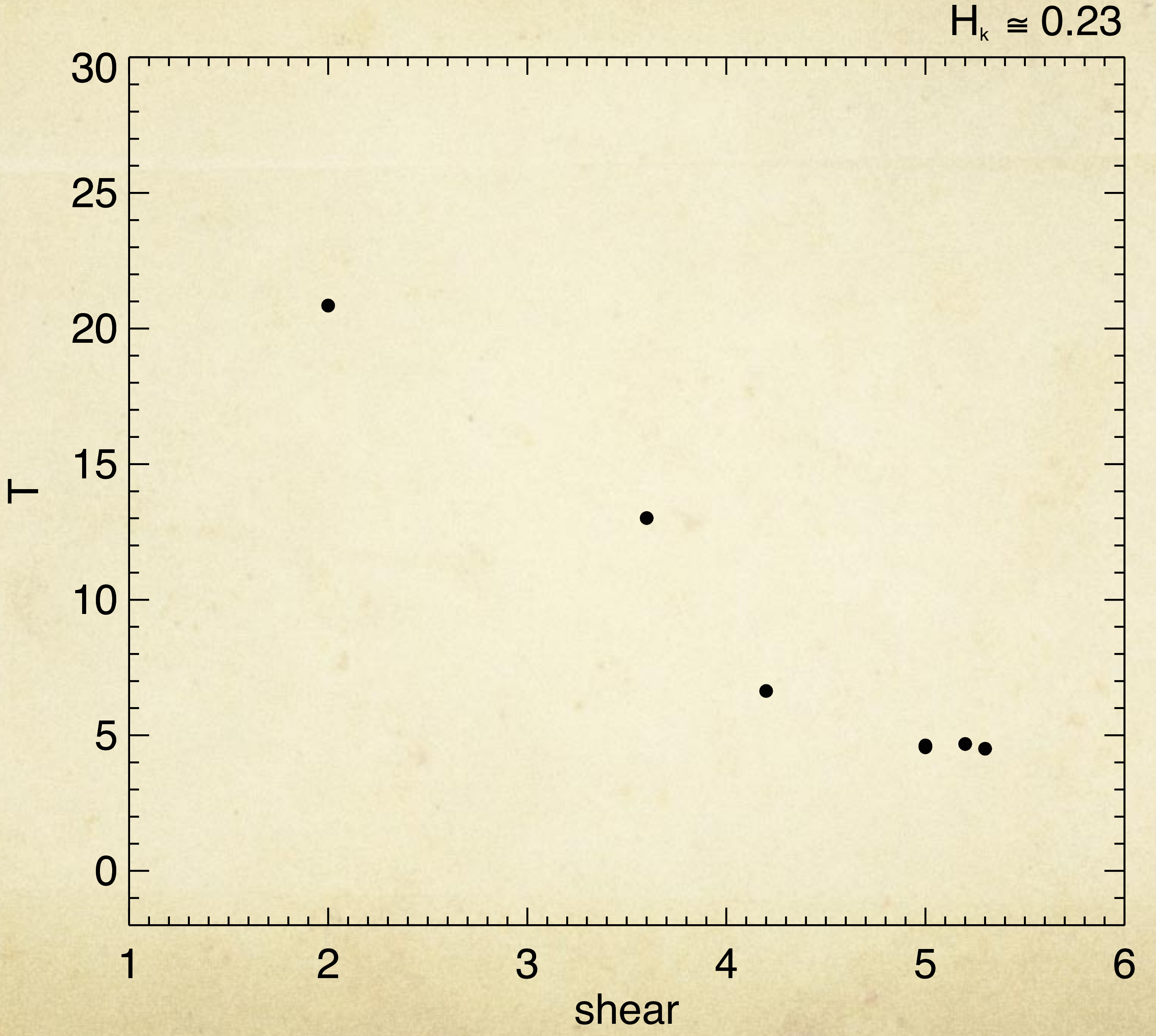
Shear = 5.2 => T = 1/ v_{max} = 1/0.21 ~ 5
Shear = 2.0 => T = 1/ v_{max} = 1/0.48 ~ 20

Periods that are compatible with those that emerge from mean-field electrodynamics, and **are not determined by the small-scale dynamo**



$< |fft(B_x(x,\ y = y_0,\ z = L_z/2,\ t))|^2 >_x$




$H_k \cong 0.23$
30
25
20
15
10
5
0
T
1
2
3
4
5
6
shear

Definition of large-scale dynamo effect

- The period of the wave component is comparable with those predicted by MFE
- The wave component **does not have a separate growth rate** from the rest of the magnetic structure.
- Both small-scale and large-scale dynamo have the same source: small-scale turbulence
- The wave component can only be unambiguously identified from the rest of the structures by its **phase coherence during the time**: all the other parts of the solution are incoherent in time

It could be better to consider a ***definition of large-scale dynamo action that considers the time–scale of evolution of the pattern, rather then one that relies on spatial scales alone***

G. Nigro, P. Pongkitiwaichakul, F. Cattaneo, S.M. Tobias, **464,** L119–L123 (2017) **MNRAS**

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Thank you for your time!